1909MNRAS..69..417P

whereas the result of using the *Nautical Almanac* data is to give

$$\Delta\alpha . \cos\delta = +\ 5''\cdot 863, \quad \Delta\delta = -\ 2''\cdot 008.$$

The differences are small, but quite sensible in comparison with some of the terms retained in calculating the nutation. There would thus seem to be a real advantage in calculating those of the day numbers which involve the aberration in the simple manner indicated above.

The question of the signs of the corrections is important, and a little intricate. That they have been rightly assigned may be seen thus. At the beginning of January the Sun passes perigee, and hence X′, Y′, Z′, of which the signs are all positive, are too large numerically. At the beginning of July the Sun passes apogee, and hence X′, Y′, Z′, of which the signs are all negative, are too small numerically. Hence the additive constant corrections required to bring the two sets together numerically must have negative signs.

Again, the position of a planet may be represented by

$$a \cos l, \quad a \sin l \cos i, \quad a \sin l \sin i,$$

and its velocity by

$$-\mathrm{V} \sin l, \quad \mathrm{V} \cos l \cos i, \quad \mathrm{V} \cos l \sin i.$$

The corresponding velocity induced in the Sun will be given by

$$m\mathrm{V} \sin l, \quad -m\mathrm{V} \cos l \cos i, \quad -m\mathrm{V} \cos l \sin i.$$

Now these components are to be added to those of the Earth relative to the Sun, and the latter are −X′, −Y′, −Z′. Hence as additive corrections to X′, Y′, Z′ they must be reversed in sign, and thus the signs given are verified.

Note on the observed time of minimum phase of an Algol Variable Star. By H. C. Plummer.

The idea of Dr. A. W. Roberts,[*] that the relative retardation of the minimum phase of an Algol variable could be used to determine the absolute dimensions of the orbit, was so ingenious that it seemed unfortunate when Father Stein [†] pointed out that the small interval of time in question must be interpreted differently. Both the dimensions of the orbit and the ratio of the masses are involved, and they cannot be separately determined from the photometric observations.

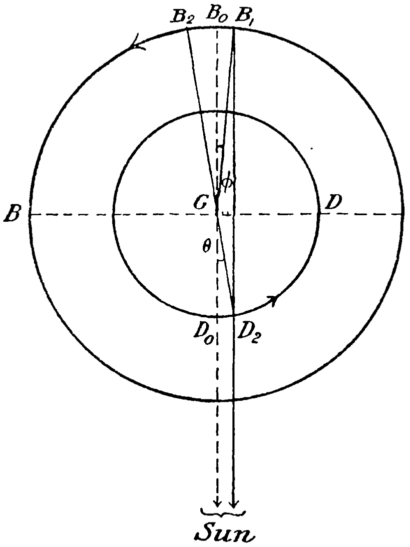
It may be worth pointing out that theoretically a knowledge of the instant of minimum phase, when used in conjunction with a spectrographic determination of the orbit of the bright component, will serve to determine both the dimensions of the orbit of the

[*] *M.N.*, lxvi. p. 123. [†] *M.N.*, lxviii. p. 490.

1909MNRAS..69..417P

dark component and the masses of the stars. For the present at least this must be regarded as a theoretical possibility only; an extreme degree of precision is demanded of the photometric and spectrographic observations, and is not yet attainable.

Let G be the centre of gravity of the system (see figure), $B_0 B$ the circular orbit of the bright component, $D_0 D$ the orbit of the dark component. Let b and d be the radii of these orbits. Let B_0, D_0 be the positions of the stars when they are in line with the Sun. Now the minimum phase will occur when the light starts from B_1 and is occulted by the dark body on reaching D_2, where B_1D_2 is parallel to $B_0 D_0$. During the passage of the light from B_1 to D_2 the bright star will have moved on to the position B_2. Let $B_0GB_1 = \phi$, and $B_0GB_2 = D_0GD_2 = \theta$. Since $B_1 D_2$ is parallel to $B_0 D_0$,



$$d \sin \theta = b \sin \phi \quad . \quad . \quad . \quad . \qquad (1)$$

and since the light-time from B_1 to D_2 is equal to the orbital time from B_1 to B_2,

$$\frac{d \cos \theta + b \cos \phi}{U} = \frac{\theta + \phi}{n} \quad . \quad . \quad . \qquad (2)$$

where U is the velocity of light and n is the orbital mean motion. Hence θ and ϕ are determined in terms of b and d.

The observed phenomenon starts from D_2 at a time θ/n after the dark body left D_0. Let B D be perpendicular to $B_0 D_0$. The observed times are to be reduced to a constant distance by referring the phenomena to the line B D. Hence the actual retardation of the minimum phase is

$$R = \theta/n - d \cos \theta/U \quad . \quad . \quad . \qquad (3)$$

But again, the radial velocity curve determined by spectrographic observations will give, *inter alia*, the instant when the

1909MNRAS..69..417P

radial velocity of the bright body is halfway between its minimum and its maximum amount, and this velocity corresponds to the body passing through the position B_0. The retardation in this observed time is clearly

$$R' = b/U,$$

so that the phenomenon now considered will follow the time when the minimum phase is observed at an interval

$$R' - R = b/U + d\cos\theta/U - \theta/n \quad . \quad . \qquad (4)$$

Since the orbital velocities are small compared with the velocity of light, θ and ϕ are small angles. Neglecting powers of θ and ϕ above the first, we obtain from (1) and (2)

$$\theta/b = \phi/d = n/U,$$

and hence

$$R = (b - d)/U \quad . \quad . \quad . \quad . \qquad (5)$$

$$R' - R = d/U \quad . \quad . \quad . \quad . \qquad (6)$$

The retardation R can be determined from the quadrant points of the light-curve, as Dr. A. W. Roberts pointed out, when both components contribute to the observed light. The observed interval $R' - R$, which can be determined whether the companion be luminous or dark, will give d, and b and d with n will give the masses of the two stars. The motion of the system as a whole relative to the Sun has been neglected. The effect of a uniform radial velocity of G, the centre of gravity, will clearly be to alter the time-scale to which the observed phenomena are referred, and unless allowed for, will cause a small error in the calculation of the masses. Of course the bearing of such a remark is purely theoretical.

The spectrographic determination of the orbit of the bright component will give the radius b, but it will by itself give no information as to d. It has been assumed that the plane of the orbit contains the line of sight, and that the luminosity of the darker component is negligible. If the lines of both components appear in the spectrum, $b \sin i$ and $d \sin i$ can be separately determined, i being the inclination of the orbit. Thus the ratio of the masses will be found, but i can only be found from the light-curve. An excellent example of this type of star is V Puppis, which Professor Pickering* announced to be a spectroscopic binary with composite spectrum, and which has been investigated as a light-variable by Dr. Roberts.† The periods found for the two variable characters appear to be totally different, and it is of urgent importance that a spectrographic investigation of the orbit of this star should be made with the greatest possible thoroughness.

So far as actual observations of Algol are concerned, it is evident that the theory of the present note cannot be applied.

* *H.C.O. Circular*, No. 14. † *Astroph. J.*, xiii. p. 181.

1909MNRAS..69..417P

For, so far from finding that the time of "mid-velocity" follows the time of light-minimum by a very short interval, Dr. Schlesinger * finds the observations of Belopolsky, Curtiss, and himself to agree at least in this, that the minimum phase lags an hour or two behind the time demanded by the velocity determinations. This suggests that the influence of tidal and rotational forces is operating to modify the figures of the stars; at any rate, the effect we have to seek is evidently masked by disturbing causes, such as have been suggested by Professor Lebedew † in a recent article.

The eccentricity of the orbit has been neglected. But in any case the star B has its mid-velocity (which must not be confused with the mean velocity, or velocity of the system) when it comes to B_0 where B_0 G is perpendicular to the line of nodes B D. Hence neither eccentricity nor inclination will affect the qualitative result and allowance for the effects of these factors will be made with sufficient accuracy by interpreting *b* and *d* as the projections on the plane containing the line of sight and the line of nodes of the actual radii vectores at B_0 and D_0. For the mean motion *n* must also be substituted the actual angular motion about G at the same points.

Note on Mr. W. S. Franks' Analysis of the Colours and Magnitudes of 3630 *Stars* (*M.N.*, lxix. p. 106). By Julia Bell, M.A.

(*Communicated by Professor Karl Pearson, F.R.S.*)

Mr. Franks' tables admit at once of treatment by the method of contingency. We have the following results: if C_2 be the coefficient of mean square contingency,

Combined galactic regions,	$C_2 = \cdot 22$
Combined non-galactic regions,	$C_2 = \cdot 18$
Total stars of both series,	$C_2 = \cdot 19$

It would appear that the relation between luminosity and colour is slightly more intense in the galactic regions.

Previous values for the contingency of magnitude and colour (Gibson and Pearson, *Monthly Notices*, lxviii. p. 419) are $C_2 = \cdot 30$ and ·27. These values are distinctly higher, but it may be noted that they were for *photographic* magnitudes, while Mr. Franks' results are for *visual* magnitudes, and contain no blue or green stars.

The percentages values of the different colours in the two groups are compared with those of stars in general by Mr. Franks on p. 107. It is interesting to note the probability that the galactic or the non-galactic stars are a "random sample" from stars in general. Applying the usual method for "goodness of fit"

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* *Publ. Allegheny Obs.*, i., No. 5. † *Astrophys. J.*, xxix. p. 101.